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# DYNAMICAL SYSTEMS MODELS FOR GROWTH OF RYEGRASS AND CLOVER

A THESIS PRESENTED IN PARTIAL FULFILMENT  
OF THE REQUIREMENTS FOR  
THE DEGREE OF MASTER OF SCIENCE IN  
MATHEMATICS AT  
MASSEY UNIVERSITY

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February 20, 1998

# Abstract

This thesis presents models formulated to describe ryegrass and clover growth independently, with the long-term goal of constructing a unified ryegrass/clover model. The purpose of this unified model will be to address the questions of co-existence of ryegrass and clover when grown together, and persistence of ryegrass in these mixed pastures.

An overview of the problem and background of the biology is provided. This may be particularly useful for the reader with no prior knowledge of ryegrass or clover biology.

A physiologically-based model for ryegrass growth is investigated. This model is a modification of that proposed by Johnson and Thornley [14] who only consider the vegetative growth phase. The modified model accounts for the reproductive growth phase. Some numerical results, with and without reproduction, are presented. These results show that increased growth occurs when reproduction is included.

A model for ryegrass growth based on tiller numbers is then investigated. This model has far fewer state variables than the above-mentioned physiologically-based model, although only vegetative growth is considered. The differential-delay equations which result from the mathematical formulation of this model are presented. Mathematical analysis of these equations reveals two steady states: a zero steady state and a finite steady state. A threshold condition that determines which of

these two steady states is eventually reached is given. The effects of harvesting the growth are also studied using numerical simulation.

Two models for clover growth, both of which are structurally similar to the second ryegrass model, are described. However the first clover model does not have any inbuilt delay mechanisms. Mathematical analysis of these models also shows the existence of two steady states: a zero steady state and a finite steady state. Again, a threshold condition determining which of these is eventually reached is given. There is little difference between the results from the two clover models, even when the numerical simulations from harvesting are considered.

Finally, a summary is given of the models studied and an indication of possible extensions to these models. A suggestion as to how a unified ryegrass/clover model might be formulated is also given.

# Acknowledgements

I would like to thank Professor Robert McKibbin for his excellent guidance, and for the enthusiasm and patience he has shown during the course of this work.

I am greatly indebted to Harry Clark, Paul Newton and Ken Louie from AgResearch Limited, Grasslands for providing me with the opportunity to work on this interesting research project.

I would like to express my sincere gratitude to Ken for his considerable help during the production of this thesis, and to Harry for his much appreciated assistance with the biology.

Many thanks to my fellow graduate students for their constant enthusiasm and support, and for convincing me to learn L<sup>A</sup>T<sub>E</sub>X.

Finally, a special thank you to my family and friends for their continuous support and encouragement.

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